M.E. 530.647 Lab 1

Revision 04

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Read the MATLAB documentation on the built-in functions `ode45()`, `odeset()`, and `plot()`.

1. Consider the first order differential equation with the following initial conditions at $t = 0$

$$\dot{y}(t) = -y(t); \qquad y(0) = y_0. \tag{1}$$

   (a) Write a matlab function `DYN1(t,y)` which accepts the scalar real arguments `t` and `y`, and returns a value of `-y`.

   (b) Use the MATLAB builtin function ODE45 to numerically integrate and plot solutions to the differential equation (1) for the time interval $t = \{0 \cdots 10\}$ seconds.

   (c) Plot your numerical solution so that you can see the individual solution points generated by ode45(), for example the following command `plot(t,x,'-o');` plots the solution as a solid line, but adds a circle at each solution point generated by ode45.

   On the same figure, plot the analytical solution to this differential equation for the same initial condition, for a time vector of 0:0.1:10.

   Generate and hand in two (or more) plots showing solutions to (1) for several different initial conditions. Annotate the plots by hand to clearly identify what they depict. Also annotate the plot with the matlab command used to generate the numerical solution.

   (d) When you zoom in on the numerical solutions, are they identical or do they differ? What is going on here?

2. Write a matlab function `SYS1(t,x)` which accepts the scalar real arguments `t` and `x`, and returns a value of $\mathtt{x}^2$. Use this function to compute numerical solutions to the equation

$$\dot{x}(t) = x^2; \qquad x(0) = x_0; \quad x_0 > 0. \tag{2}$$

   (a) Simulate using `ODE45`. Print, annotate, and hand-in solutions for at least two initial conditions.

   (b) What is the analytical solution to this ODE?

   (c) Do solutions exist?

   (d) Do solutions exist for all time?

   (e) If solutions exist, are they unique?

   (f) How do the numerical `ODE45` solutions compare to your analytical solution? Hand in a plot of the difference between the numerical and analytical solutions.

3. Write a matlab function `SYS2(t,x)` which accepts the scalar real arguments `t` and `x`, and returns a value of $-|\mathtt{x}|^{\frac{1}{2}}$. Use this function to compute numerical solutions to the equation

$$\dot{x}(t) = -|x|^{\frac{1}{2}}; \qquad x(0) = x_0; \quad x_0 < 0. \tag{3}$$

   (a) Simulate using `ODE45`. Print, annotate, and hand-in solutions for at least two initial conditions.
   (b) What is the analytical solution to this ODE?
   (c) Do solutions exist?
   (d) Do solutions exist for all time?
   (e) If solutions exist, are they unique?
   (f) How do the numerical `ODE45` solutions compare to your analytical solution? Hand in a plot of the difference between the numerical and analytical solutions.

Hand in *ANNOTATED* printouts of your plots and printouts of your m-files. Hand in your m-file functions on a floppy disk or email them to me as ZIP file attached to your email. Put "530.647 LAB #1 m-files from YOUR FULL NAME" in the subject line.

Check to verify that the files you hand in run. If your matlab functions call custom matlab m-files that you have written (for this course or otherwise) be sure to include *all* necessary files.