M.E. 530.647 Lab 2

Revision 01

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https://dscl.lcsr.jhu.edu/courses/530-647-adaptive-systems-fall-2017

1. Simulate the performance of a stable adaptive parameter estimator for the scalar linear algebraic system

$$y(t) = \theta u(t) \tag{1}$$

given by

$$\begin{aligned} \dot{\hat{\theta}} &= -\gamma\, u(t)\, \Delta y(t) \\ &= -\gamma\, u(t)\, (\hat{y}(t) - y(t)) \\ &= -\gamma\, u(t)\, (\hat{\theta} u(t) - y(t)) \end{aligned} \tag{2}$$

where $\Delta\theta = \hat{\theta} - \theta$, $\hat{y}(t) = \hat{\theta} u(t)$, $\gamma = 1$, $\theta = 3$ and $u(t) = 1$.

   (a) Write a function `adapt1`$(\mathtt{t}, \hat{\theta})$ which accepts the scalar real argument `t` and the scalar $\hat{\theta}$ and returns a *result* value of

   $$\dot{\hat{\theta}} = -\gamma\, u(t)\, \Delta y(t). \tag{3}$$

   Use this function to simulate and plot solutions to the system. You will likely need to increase the default accuracy of the ODE solution, for example: `opt = odeset('RelTol',1e-6); [t theta] = ode45(@adapt1,[0 10], 0,opt);`

   (b) Construct a Lyapunov function candidate for this system. Write the mathematical expression for your proposed Lyapunov function. Write a function `lyap1`$(\mathtt{t}, \hat{\theta})$ which evaluates the Lyapunov function. Plot values of the Lyapunov function and its time derivative for solutions to your system.

   (c) Do you observe that $\lim_{t\to\infty} \hat{\theta}(t) = \theta$?

   (d) Simulate, print, and annotate the solutions to this system for two different initial conditions and/or different γ values. For each case, plot (a) the solutions to this system versus time and (b) the value of the Lyapunov function and (c) the time derivative of the Lyapunov function.

   (e) Comment on the behavior of each signal, and compare the observed behavior to that which you expect based upon your analysis of this problem. Explain any discrepancies.

2. Repeat the previous question for the case $u(t) = sin(t)$. Use function names `adapt2`$(\mathtt{t}, \hat{\theta})$ and `lyap1`$(\mathtt{t}, \hat{\theta})$

3. Simulate the performance of a stable adaptive parameter estimator for the vector linear algebraic system

$$y(t) = \theta^{\mathrm{T}} u(t) \quad y \in \mathbb{R}^1, \;\; \theta, u(t) \in \mathbb{R}^2 \tag{4}$$

given by

$$\begin{aligned}\dot{\hat{\theta}} &= -\gamma\ u(t)\ \Delta y(t) \\ &= -\gamma\ u(t)\ (\hat{y}(t) - y(t)) \\ &= -\gamma\ u(t)\ (\hat{\theta}^T u(t) - y(t))\end{aligned} \tag{5}$$

where $\Delta\theta = \hat{\theta} - \theta$, $\hat{y}(t) = \hat{\theta}^T u(t)$, $\gamma = 1$, $\theta = [3; 5]$ and $u(t) = [1; 0.5]$.

(a) Write a function `adapt3`$(\texttt{t}, \hat{\theta})$ which accepts the scalar real argument `t` and the scalar $\hat{\theta}$ and returns a *result* value of

$$\dot{\hat{\theta}} = -\gamma\ u(t)\ \Delta y(t) \tag{6}$$

(b) Construct a Lyapunov function candidate for this system. Write the mathematical expression for your proposed Lyapunov function. Write a function `lyap3`$(\texttt{t}, \theta)$ which evaluates the Lyapunov function. Plot values of the Lyapunov function and its time derivative for solutions to your system. Plot the value of the Lyapunov function versus time.

(c) Do you observe that $\lim_{t\to\infty} \hat{\theta}(t) = \theta$?

(d) Simulate, print, and annotate the solutions to this system for two different initial conditions and/or different γ values. For each case, plot (a) the solutions to this system versus time and (b) the value of the Lyapunov function and (c) the time derivative of the Lyapunov function.

(e) Comment on the behavior of each signal, and compare the observed behavior to that which you expect based upon your analysis of this problem. Explain any discrepancies.

4. Construct a continuous function $u(t)$, $u : \mathbb{R}^1 \to \mathbb{R}^2$, which satisfies the persistent excitation condition given in class. Show your work verifying that your $u(t)$ has the required properties.

   Repeat the previous question for the case of your new $u(t)$.

   Use function names `adapt4`$(\texttt{t}, \hat{\theta})$ and `lyap3`$(\texttt{t}, \delta\theta)$

Hand in *ANNOTATED* printouts of your plots and printouts of your m-files. Hand in your m-file functions by emailing them to me as ZIP file attached to your email. Put "530.647 LAB #2 m-files from YOUR FULL NAME" in the subject line.

Check to verify that the files you hand in run. If your matlab functions call custom matlab m-files that you have written (for this course or otherwise) be sure to include *all* necessary files.