# M.E. 530.647 Problem Set 3

Revision 02

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Refer to your class notes and [1]. Where appropriate, be sure clearly identify and state the following (*i*) plant, (*ii*) reference model, (*iii*) task, (*iv*) error coordinates, (*v*) control law, (*vi*) controlled plant, (*vi*) error system, (*vii*) stability analysis, (*vii*) parameter update law, (*viii*) stability conclusions.

1. Design and prove the stability of an adaptive identifier for the system

$$\dot{x}(t) = Ax(t) + Bu(t) \tag{1}$$

where $A \in \mathbb{R}^{n \times n}$ and $B \in \mathbb{R}^{n \times m}$ are unknown constant matricies, the signals $x(t)$ and $u(t)$ are accessible, $A$ is Hurwitz, and $u(t)$ is bounded.

Be sure to:

(a) Define the plant.

(b) Define and clearly identify all components of the adaptive identifier.

(c) Define the error coordinates.

(d) Show the full resulting error systems.

(e) Show that your proposed system is stable. Be sure to clearly define your Lyapounov function, and all steps of the stability proof. Clearly identify (and justify) the conclusions of your stability proof.

2. Consider the model reference adaptive control problem for the linear algebraic plant

$$y_p = a_p u(t) \qquad a_p \neq 0 \tag{2}$$

and the reference model

$$y_m = a_m r(t) \tag{3}$$

where $r(t)$ is bounded and continuous. Design a differentiator-free control $u(t)$ such that (*i*) all signals are bounded, and (*ii*) $\lim_{t\to\infty}(y(t) - y_m(t)) = 0$. Show that your proposed method accomplishes these objectives. This is a variation on a problem from [1].

Consider the cases where

(a) $|a_p|$ is unknown but $sgn(a_p)$ is known

(b) $a_p$ is unknown.

# References

[1] K.S. Narendra and A. Annaswamy. *Stable Adaptive Systems*. Dover Publications, NY, 2005.