M.E. 530.647 Problem Set 4

Revision 01

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Refer to your class notes and [1]. Where appropriate, be sure clearly identify and state the following (i) plant, (ii) reference model, (iii) task, (iv) error coordinates, (v) control law, (vi) controlled plant, (vi) error system, (vii) stability analysis, (vii) parameter update law, $(viii)$ stability conclusions.

1. Consider the *indirect* model reference adaptive control problem for the plant

$$\dot{x}(t) = ax(t) + bu(t) \tag{1}$$

with constant unknown scalar parameters a and b, where $b \neq 0$, and

$$\dot{x}_m = a_m x_m(t) + b_m r(t) \quad a_m < 0 \tag{2}$$

is the reference model. Design a differentiator-free *indirect* model reference adaptive control $u(t)$ such that (i) all signals are bounded and (ii) $\lim_{t\to\infty}(x(t) - x_m(t)) = 0$. Show that your proposed method accomplishes these objectives.

2. You have been asked to design an automotive cruise control system for use on all makes and models of GM automobiles. The vehicle performance engineers have determined that the equation of motion for a car on flat dry pavement takes the form

$$m\dot{v}(t) = -c_d \, v(t)|v(t)| + g \, u(t) \qquad c_d > 0, g > 0 \tag{3}$$

where m is the mass of the car ($kilograms$), $v(t)$ is its velocity ($meters/second$), $c_d \, v(t)|v(t)|$ is the force on the vehicle due to wind drag, c_d is the wind drag coefficient ($Newton \; sec^2/m^2$), $u(t)$ is the position of the gas-pedal ($degrees$), and g is the ratio of gas-pedal degrees to axial motor/wheel force (Newton/degree). The constants m, g, and c_d are all positive, but different car models have differing values. Observe that the equation of motion can be rewritten in the form

$$\dot{v}(t) = -m^{-1} c_d \, v(t)|v(t)| + m^{-1} g \, u(t) \tag{4}$$

or

$$\dot{v}(t) = -\beta \, v(t)|v(t)| + \alpha \, u(t) \tag{5}$$

where α and β are positive constants. The GM human-factors engineers have provided the reference-model

$$\dot{v}_m(t) = a_m v_m(t) + r(t) \qquad a_m < 0 \tag{6}$$

where $r(t)$ is the (smoothly time varying) desired vehicle velocity specified by the driver.

(a) Design a differentiator-free *direct* model-reference adaptive controller $u(t)$ such that (i) all signals are bounded and (ii) $\lim_{t\to\infty}(v(t) - v_m(t)) = 0$. Assume that $m > 0$, $g > 0$, and $c_d > 0$ are all positive, but of unknown magnitudes. Show that your proposed method accomplishes these objectives.

(b) Is your controller structurally able to estimate precise values for the individual constant plant parameters m, g, and c_d? Why or why not?

References

[1] K.S. Narendra and A. Annaswamy. *Stable Adaptive Systems*. Dover Publications, NY, 2005.