M.E. 530.647 Problem Set 5

Revision 02

Louis L. Whitcomb*
Department of Mechanical Engineering
G.W.C. Whiting School of Engineering
Johns Hopkins University
https://dscl.lcsr.jhu.edu/home/courses/530-647-adaptive-systems-fall-2017

1. Consider the case a pendulum with massless link of length l meters, posessing a point mass of mass m Kg, and a perfect torque motor which can apply torque $\tau(t)$ at the pendulum's frictionless joint.
   (a) Draw a diagram of the system. Clearly indicate a coordinate system in which the joint angle of the pendulum is measured, in units of radians, as the clockwise rotation from the pendulum's stable rest position.
   (b) Derive the equations of motion for the pendulum.
   (c) Consider the problem of non-adaptive trajectory tracking for this plant. Assume you are given a smooth reference signal $q_d(t)$ and its time derivatives $\dot{q}_d(t)$ and $\ddot{q}_d(t)$.
   Assume that all plant parameters are known. Assume full state accessibility.
      i. State the error coordinates of this task.
      ii. Design an exactly linearizing non-adaptive control law that accomplished the following design objectives for trajectory tracking:
         - All signals remain bounded.
         - $\lim_{t\to\infty}(q(t) - q_d(t)) = 0$.
         - $\lim_{t\to\infty}(\dot{q}(t) - \dot{q}_d(t)) = 0$.
      iii. Show the dynamical equation of the controlled plant.
      iv. Prove mathematically that the closed loop system accomplishes the stated objectives.
   (d) Consider the problem of adaptive trajectory tracking for this plant. Assume you are given a smooth reference signal $q_d(t)$ and its time derivatives $\dot{q}_d(t)$ and $\ddot{q}_d(t)$.
   Assume that the plant mass and inertial parameters are unknown. Assume that the plant kinematic parameters are known. Assume full state accessibility.
      i. State the error coordinates of this task.
      ii. Design an exactly linearizing adaptive control law that accomplished the following design objectives for trajectory tracking:
         - All signals remain bounded.
         - $\lim_{t\to\infty}(q(t) - q_d(t)) = 0$.
         - $\lim_{t\to\infty}(\dot{q}(t) - \dot{q}_d(t)) = 0$.
      iii. Show the dynamical equation of the controlled plant.
      iv. Prove mathematically that the closed loop system accomplishes the stated objectives.

2. The equations of motion of a rigid-body open kinematic chain in local joint coordinates, $q(t) \in \mathbb{R}^n$, in the presence of external forces arising from (i) the earth's gravitational potential, $g(q)$, (ii) independently controlled torque actuators, $\tau(t) \in \mathbb{R}^n$, can be written in the form

$$\tau \quad = \quad M(q)\ddot{q} + C(q, \dot{q})\dot{q} + g(q) \tag{1}$$

where $M(q)$ and $C(q, \dot{q})$ are, respectively, the standard inertial and coriolis matrices.

(a) Use the Lagrange-Euler method to derive the equation of motion for a two-degree of freedom revolute-revolute arm shown in Figure 1.

(b) Factor the resulting expression into the form (1).

(c) Show by direct computation that $\dot{M} - 2C$ has a skew-symmetric form.

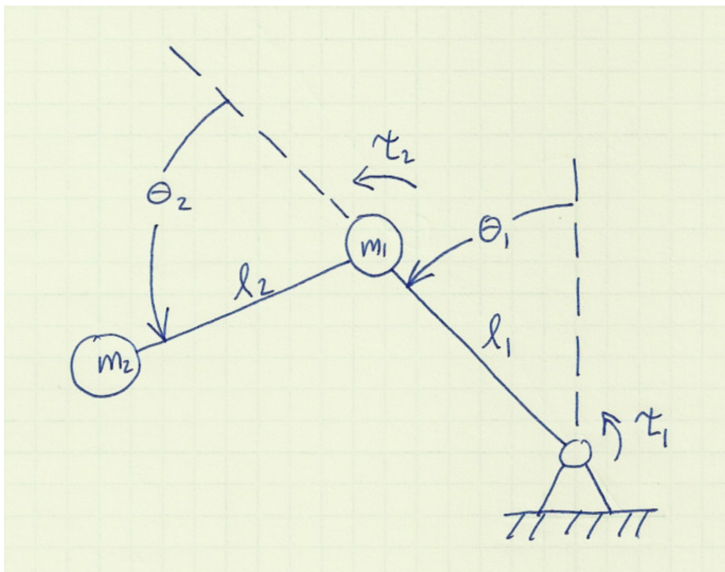

Figure 1: RR Arm: A planar arm with two point-masses and two revolute joints.

3. In class we examined non-adaptive *exact-linearization* ~~sliding-mode~~ tracking control for robot arm plants of the form (1). Derive the *adaptive* extension of this approach for robot arm plants.

Assume you are given a smooth reference signal $q_d(t)$ and its time derivatives $\dot{q}_d(t)$ and $\ddot{q}_d(t)$.

Assume full state accessibility. Assume that plant kinematic parameters are known, but that inertial parameters are unknown.

(a) Define the class of plants.

(b) State the error coordinates of this task.

(c) Design a globally stable adaptive control law that accomplished the following design objectives for trajectory tracking:
   - All signals remain bounded.
   - Asymptotically exact trajectory tracking is achieved.

(d) Show the dynamical equation of the controlled plant.

(e) Prove mathematically that the closed loop system accomplishes the stated objectives.

In your proof, be sure to explicitly show each of your conclusions holds — i.e. give an explicit mathematical argument for each and every point of your conclusions.