

# M.E. 530.647 Problem Set 6

Revision 01

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<https://dscl.lcsr.jhu.edu/home/courses/530-647-adaptive-systems-fall-2017>

1. Determine if the following system are (or are not) positive real (PR) or strictly positive real (SPR).

(a)  $A = [0 \ 1; -1 \ -2]$ ;  $b = [-1; 2]$ ;  $c = [0 \ 3]$

(b)  $A = [0 \ 1; -1 \ -2]$ ;  $b = [0; 1]$ ;  $c = [1 \ 0]$

2. Consider SISO LTI plants of the form

$$\begin{aligned}\dot{x}_p(t) &= A_p x_p(t) + b_p u(t) \\ y_p(t) &= h^T x_p(t).\end{aligned}\tag{1}$$

(a) State the non-adaptive observer problem for this class of plants.

(b) State the adaptive observer problem for this class of plants.

(c) For this class of plants, give the analytical development of a a stable non-adaptive observer, and show that this observer results in the property  $\lim_{t \rightarrow \infty} \Delta x(t) = 0$  and  $\lim_{t \rightarrow \infty} \Delta y(t) = 0$

3. Consider the non-minimal modified adaptive observer proposed in Section 4.3.4 of [1] for a SISO LTI plants of the form

$$\begin{aligned}\dot{x}_p(t) &= A_p x_p(t) + b_p u(t) \\ &= K_p x_p(t) + g_p y_p + b_p u(t) \\ y_p(t) &= h^T x_p(t).\end{aligned}\tag{2}$$

(a) For this class of plants, give the analytical development of a stable adaptive observer, and show that this observer results in the property  $\lim_{t \rightarrow \infty} \Delta y(t) = 0$ .

(b) Can this system be modified to include a scalar-valued or symmetric positive definite matrix-valued adaptation gain for the parameter adaptation? Show why or why not.

4. (optional) Construct a numerical simulation of an adaptive observer of the form from Question 3 for a plant of the form (2) where  $K_p = [-1 \ 1; -1 \ 0]$ ;  $g_p = [-1 \ -1]^T$ ;  $b_p = [1 \ 0]^T$ ;  $h = [1 \ 0]^T$ ;  $p = [b_p^T g_p^T]^T$ . Use a bounded time-varying  $u(t)$ .

(a) Plot and annotate graphs of (at least)

i.  $y_p(t)$ ,  $\hat{y}(t)$ , and  $\Delta y(t)$ .

ii.  $p$ ,  $\hat{p}(t)$ , and  $\Delta p(t)$ .

(b) Discuss the observed convergence properties of these signals.

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## References

- [1] K.S. Narendra and A. Annaswamy. *Stable Adaptive Systems*. Dover Publications, NY, 2005.